CYCLIC PLASTICITY OF A CRACKED STRUCTURE SUBJECTED TO MIXED MODE LOADING

Sylvie Pommier¹, a

¹LMT-Cachan, 61 avenue du Président Wilson, 94235 Cachan, FRANCE
apommier@lmt.ens-cachan.fr

Keywords: Fatigue, mixed mode crack propagation, plasticity, crack deflection.

Abstract. Cyclic plasticity in the crack tip region is at the origin of various history effects in fatigue. For instance, fatigue crack growth in mode I is delayed after the application of an overload because of the existence of compressive residual stresses in the overload’s plastic zone. Moreover, if the overload’s ratio is large enough, the crack may grow under mixed mode condition until it has gone round the overload’s plastic zone. Thus, crack tip plasticity modifies both the kinetics and the crack’s plane. Therefore modeling the growth of a fatigue crack under complex loading conditions requires considering the effects of crack tip plasticity. Finite element analyses are useful for analyzing crack tip plasticity under various loading conditions. However, the simulation of mixed mode fatigue crack growth by elastic-plastic finite element computations leads to huge computation costs, in particular if the crack doesn’t remain planar. Therefore, in this paper, the finite element method is employed only to build a global constitutive model for crack tip plasticity under mixed mode loading conditions. Then this model can be employed, independently of any FE computation, in a mixed mode fatigue crack growth criterion including memory effects inherited from crack tip plasticity. This model is developed within the framework of dissipative processes and includes internal variables that allow modeling the effect of internal stresses and to account for memory effects. The model was developed initially for pure mode I conditions. It was identified and validated for a 0.48%C carbon steel. It was shown that the model allows modeling fatigue crack growth under various variable amplitude loading conditions [1]. The present paper aims at showing that a similar approach can be applied for mixed mode loading conditions so as to model, finally, mixed mode fatigue crack growth.

Introduction

Let us briefly explain what has already been done for mode I fatigue crack growth under variable amplitude loadings [1,2]. The fatigue crack growth model is based on the assumption that the fatigue crack growth rate is function of crack tip plasticity. Therefore the model is divided into two parts. The first part provides the fatigue crack growth rate as a function of the rate of variation of a measure of plastic deformation in the crack tip region. The second part allows predicting the rate of variation of that measure according to the loading conditions and of the values of a set of internal variables introduced so as to model history effects.

This second part of the model can be considered as an elastic-plastic constitutive model for the crack tip region. It was developed within the framework of dissipative processes as follows: first of all, it is assumed that the displacement field in the crack tip region is partitioned into elastic and plastic parts (Eq. 1). Each part is also assumed to be the product of an intensity factor and of a reference field function only of spatial coordinates. The intensity factor of the elastic field is, classically, the stress intensity factor $K_I$, while the intensity factor of the plastic part, is labeled as $\rho$ the plastic strain intensity factor.

$$u(\mathbf{x},t) = \tilde{K}_I(t) u_I(\mathbf{x}) + \rho(t) u_\rho(\mathbf{x})$$

(1)
It is always possible to perform an approximation such as that in Eq. 1 using a mathematical transform such as the Karhunen-Loève transform for instance. However, in the present case, it was possible to justify [2] that the “elastic” spatial field \( \bar{u}^e_I(x) \) is the solution of an elastic FE computation using boundary conditions such as that \( K_I = 1 \) and that \( \bar{u}^e_I(x) \) is the displacement field around an edge dislocation aligned with the crack plane and with a Burgers vector equal to \( l \) [3].

The approximation in Eq. 1 is performed at each step of an FE computation using a post-treatment routine based on the least square method. By comparing the computed displacement field and the approximated one, it is also possible to show that the least square error remains below 10% under reasonable loading conditions [2].

Thus, the plastic strain intensity factor \( \rho \) is a global measure of plastic deformation in the crack tip region. The finite element method and the post-treatment routine are employed so as to generate evolutions of \( \rho \) under various loading conditions (Fig. 1). Then an empirical model, developed within the framework of dissipative processes, is associated to these evolutions and allows predicting \( d\rho/dt \) as a function of \( dK_I/dt \) and of a set of internal variables [2]. As a matter of fact, at each load’s reversal (Fig. 1), it is observed that there is a domain within which no variation of \( \rho \) is observed. Since \( \rho \) is a measure of crack tip plasticity this domain (E in Fig. 1) can be considered as an elastic domain for the crack tip region. This domain moves (Fig. 1). Therefore a first internal variable should be introduced so as to define the position of E. Since, its size varies, a second internal variable is introduced so as to define its size.

Then, empirical evolution laws are also introduced for these internal variables functions of the rate of variation of \( \rho \) and of the crack growth rate. These evolution laws are identified using FE analyses. Finally, a fatigue crack growth criterion is introduced which states that the crack growth rate is function of the rate of variation of the plastic strain intensity factor (Eq. 2, for instance).

\[
\frac{da}{dt} = a\left|\frac{d\rho}{dt}\right| \tag{2}
\]

This approach is operational for mode I fatigue crack growth, and, out of its numerical efficiency, it has other advantages such as avoiding cycle counting.

**Mixed mode loading conditions.** So as to generalize this approach to mixed mode loading conditions, the partition hypothesis in Eq. 1 should be enriched (Eq. 3). Elastic and plastic parts are introduced for each mode, and each part is the product of a known spatial field and of an intensity factor. The global variables are now, the mode I and II stress intensity factors \( K_I \) and \( K_{II} \) and the mode I and II plastic strain intensity factor \( \rho_I \) and \( \rho_{II} \).

\[
u(x) = \tilde{K}_I \bar{u}^e_I(x) + \tilde{K}_{II} \bar{u}^e_{II}(x) + \rho_I \bar{u}^e_I(x) + \rho_{II} \bar{u}^e_{II}(x) \tag{3}
\]
The finite element method is then employed to generate evolutions of $\rho_I$ and $\rho_{II}$ under various loading conditions. For instance, the cracked structure is preloaded, then partially unloaded and from this point various loading path are examined. For each loading path the evolutions of $\rho_I$ and $\rho_{II}$ are calculated.

**Elastic domain.** Using such computations, an elastic domain is determined from FE results as the domain within which $\rho_I$ and $\rho_{II}$ remain close to zero.

First of all, an analytical expression for this domain is proposed which fits well the finite element results. This expression is, more or less, a generalization of the Von Mises yield criterion to the crack tip region. It is proposed that the yield criterion of the crack tip region is a critical elastic distortional energy in this region. Therefore the yield point for any mixed mode loading condition can be deduced from that obtained under pure mode I loading condition $K_{IY}$ [4]. The analytical expression (Eq. 4) fits well the numerical results as it can be seen in Fig. 2.

$$\frac{d(K_{IY}^2)}{K_{IY}^2} + \frac{d(K_{IIY}^2)}{K_{IIY}^2} = 1,$$ with:  
$$K_{IIY}^2 = K_{IY}^2 \frac{7 - 16\nu + 16\nu^2}{19 - 16\nu + 16\nu^2}$$  
(4)

Secondly, it appears that it might be possible to represent the history of prior plastic deformation in the crack tip region, merely by a displacement and a size evolution of that elastic domain (Fig. 2). A distortion of that domain is also observed but it remains moderate and will be neglected.

**Flow direction.** For each computation the flow direction $\phi$ was computed as indicated in Eq. 5 and reported in Fig. 2. A sudden modification of the flow direction is found at the vertex of the yield surface.

$$\tan \phi = \frac{d\rho_I}{d\rho_{II}}, \quad \text{and} \quad \tan \theta = \frac{dK_{IY}^2}{dK_{IIY}^2}$$  
(5)
Finally it is also possible to compare the flow directions in both cases and their evolutions according to the loading direction $\theta$ (Eq. 5). The flow direction, except for the sign, is more or less constant whatever the loading direction (Fig. 3). However there is a strong dependency of the flow direction $\phi$ to the history since the flow direction computed in case (a) of Fig. 2 is different from that in case (b) of Fig. 2.

At this stage it becomes possible to build an empirical plasticity model for the crack tip region. This model includes internal variables such as the size and the position of the yield surface for crack tip cyclic plasticity, it contains also a yield criterion (Eq. 4) and a flow rule which is function of history. This last point remains to be clarified.

Conclusions

An approximation is proposed for the displacement field in the crack tip region under mixed mode loading conditions which enables to envision crack tip plasticity at the global scale through the evolutions of two plastic strain intensity factors $\rho_I$ and $\rho_{II}$. On the basis of this hypothesis and using finite element computations, it was possible to exhibit an elastic domain for the crack tip region and to propose a yield criterion based on a critical variation of the elastic distortional energy in that region that fits well the FE results. Finally it is also possible to analyze the flow direction at the global scale and to correlate it to the loading direction and to the loading history.

In the future, a crack growth criterion (kinetics and plane) should also be defined so as to obtain a mixed mode fatigue crack growth model including history effects inherited from crack tip plasticity.

References