Gurson’s plasticity coupled to damage as a CAP model for concrete compaction in dynamics

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Abstract:

In this work we propose an original way to obtain a cap model for the concrete behaviour under dynamic loading such as impacts or explosions. For these kinds of solicitations, we have an important spherical stress (pressure) in the material that induces a variation of the volume fraction of voids (porosity) in it. This phenomenon, well known in soils, is generally modelled with a CAP model (DiMaggio and Sandler 1971). In our work we propose to model this decrease of porosity (compaction) but also the plastic strains in compression and cracking in tension. Recently, new dynamic experiments performed on confined concrete (to obtain compaction), highlighted a significant effect of the strain rate on the spherical behaviour of concrete. Following these experimental observations, a viscoplastic and visco-damage model was developed. This model is based on Perzyna viscoplasticity associates with a modified Gurson yield function and on a visco-damage model. The model presented was implemented in the finite element code LS-DYNA3D. Simulations of a test carried out on a structure allow us to validate the numerical implementation, as well as the model for fast dynamic loading.

Introduction

In the present paper, which is a sequel of recent research efforts carried out at LMT-Cachan (Burlion et al. 2000) and (Gatuingt and Pijaudier-Cabot 2002), we present a constitutive law for concrete in dynamics based on viscoplasticity combined with rate dependent continuum damage. This model is able to represent the entire phenomenon observed during an impact on a slab. In this range of loading, the behavior of concrete is generally described by means of the plasticity theory where the spherical and the deviatoric responses are considered separately. The major differences between different models are found in the way both responses are linked together. In most approaches, the deviatoric part is modelled with a plasticity-based, or viscoplastic model, while the spherical part, also called "equation of state", is fitted with a cap model (Lubarda et al. 1996, Le Vu 1998). Indeed, it is possible to

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decompose these solicitations in three types for high thickness structures (Figure 1) (Zukas 1992): in a first time, concrete experiences a very high state of triaxial compression in the “input” face, which leads to compaction. This compaction results from the crushing of the material porosity and leads to an increase of the mass and of the elastic moduli. Such situations are generally found in Military applications or in studies for safety of buildings (power plants) regarding to accidental internal loading or external loading (plane crash). At the same time, this phenomenon is accompanied with large deformations leading to spalling of the structure. In a second time, shear loadings occur as well as compression with low confinement at the time of striker penetration in the structure: this phenomenon is called tunneling. Finally, traction loadings occur on the back face of the slab due to the reflection of the compressive waves: this leads to scabbing.

The phenomenological models are usually capable to describe correctly two of the three phenomena. Particularly, the damage models (Mazars 1984, La Borderie 1987, Pijaudier-Cabot and Bazant 1987, Mazars and Pijaudier-Cabot 1989, de Borst et al. 1993) allow to correctly describe the concrete under traction. Moreover, plasticity likes models (Dragon and Mroz 1979, Chen and Buyukozturk 1985) are able to reproduce the behavior of the concrete under shear and compression with low confinement loadings. On the other hand, we have a lack of modelisation for the compaction part. Some models (Drucker and Prager 1952, Azzouz 1995, Lubarda et al. 1996) adapted for rocks and soils can be extrapolated to concrete but they are most of time very hard to implemented.

In the model presented here, the three mechanisms will be combined: compaction which is modelled with a homogenisation technique, tensile damage which is described with a rate dependent damage model and compression failure which is modelled with viscoplasticity combined to damage. This relation is restricted to cases with moderate strain rates in concrete, corresponding typically to explosions and impacts of projectiles at a velocity less than 350 m/s, yielding a hydrostatic pressure in the material, which is less than 1 GPa. A finite element
example will be presented in order to illustrate the capabilities of the proposed model.

**Constitutive Equations**

There are two mechanisms that induce a variation of the elastic moduli of the material: microcracking in tension and the crushing of the cement or mortar matrix in hydrostatic compression. For the first phenomenon, we use a classical rate dependent damage model (Dubé *et al.* 1996), which uses two damage variables in order to provide a realistic response of the material in uniaxial compression while preserving a good description of what occurs in tension, which is a characteristic of damage models. Rate effects are necessary in order to represent dynamic experiments (mostly dynamic tensile tests). In addition, rate dependency preserves well posedness of the equations of motion when strain softening occurs (Sluys 1993).

The second phenomenon is captured by using a modified Gurson yield function (Gurson 1977, Needleman and Tvergaard 1984) in which the porosity of the material \( f^* \) is governed by the plastic flow, keeping in mind that in practice the material is always compacted (the porosity always decreases because, in tension, cracking occurs first, before the material porosity increases). In addition, we use a homogenisation technique to have elastic properties of material functions of the variation of porosity, especially during hydrostatic loadings. The method due to Mori-Tanaka is selected because it provides explicit expressions of the shear and bulk moduli of (uncracked) concrete function of the porosity. The rate dependent effects are necessary in order to capture the increase of resistance in triaxial compression observed experimentally in dynamic (Gatuingt 1999).

These two mechanical effects are combined in the final relationships that relate the stresses to the elastic strains:

\[
\sigma_{ij} = (1 - D) \left[ Ke_{kk} \delta_{ij} + 2G \left( e_{ij}^e - \frac{1}{3} e_{kk}^e \delta_{ij} \right) \right]
\]

with

\[
\dot{e}_{ij}^e = \dot{e}_{ij} - \dot{e}_{ij}^{vp}
\]

where \( \dot{e}_{ij}^{vp} \) is the viscoplastic strain rate.

**Viscoplastic model**

The experiment developed within the French network GEO showed that there was a dependence of the loading rate on the curve relating the volumetric strain to the hydrostatic stress (Gatuingt 1999), in addition to permanent plastic strains and to the material compaction which induces an increase of the bulk and shear moduli of
the material. It is thus pertinent to implement a viscoplastic model that will be coupled in the next section to the above damage model.

The viscoplastic strains are obtained following Perzyna’s approach:

\[
\dot{\varepsilon}_{ij}^{vp} = \dot{\lambda} \frac{\partial F_{NT}}{\partial \sigma_{ij}}
\]  

(3)

\( F_{NT} \) is the modified Gurson’s yield function proposed by Needleman and Tvergaard 1984:

\[
F_{NT}(\sigma_{ij}, \sigma_M, f^*) = \frac{3J_2}{2\sigma_M} + 2q_1f^* \cosh(\frac{I_1}{2\sigma_M}) - (1 + (q_2f^*\gamma)^2) = 0
\]  

(4)

where \( I_1 \) is the first invariant and \( J_2 \) is the second invariant of the stress tensor, \( \sigma_M \) is the elastic limit of the material without pores (the matrix), \( f^* \) is the porosity, \( q_1, q_2, \) and \( q_3 \) are material parameters. Figure 2 shows the evolution of the yield surface versus porosity. For symmetric reasons, only a quarter of the \((J_2/\sigma_M, \sqrt{J_2/\sigma_M})\) plane is represented. We can notice that the elastic domain increases when the porosity decreases, which is really close to reality, and when it is equal to zero we obtain the Von Mises criterion. We use this yield surface because for \( f^* \neq 0 \) we have a kind of cap model. This is this cap on the hydrostatic stress axis that will produce the evolution of the porosity.

Figure 2: Representation of the Gurson’s yield function for \( q_1 = q_2 = q_3 = 1 \)
The work hardening assumption for a material with voids was given by Needleman and Tvergaard (1984):

\[ \sigma_y e_{\bar{ij}} = (1 - f^*) \sigma_M e_M \]  

(5)

with

\[
\begin{align*}
\varepsilon_M &= \frac{\sigma}{E} \quad \text{if } \sigma \leq \sigma_y \\
\varepsilon_M &= \frac{\sigma_y}{E} \left( \frac{\sigma}{\sigma_y} \right)^n \quad \text{if } \sigma > \sigma_y
\end{align*}
\]

(6)

where \( \sigma_y \) is the elastic strength of the matrix (without pores) and \( n \) the hardening exponent. Same as in the Gurson’s model (Gurson 1977), the evolution of the porosity \( f^* \) is related to the irreversible volumetric strain (Burlion et al. 2000). We had a parameter \( k \) to the relation proposed by Burlion et al. (2000) in order to adjust the “velocity” of the pores crushing:

\[ \dot{f}^* = (1 - f^*) \cdot f^* \cdot \dot{e}_{\text{irr}}^{kk} \]  

(7)

Figure 3 shows the evolution of the porosity versus the irreversible part of the volumetric strain. When the irreversible volumetric strains increase, the porosity decreases and tends to zero. In the same time, the modified Gurson’s criterion tends to the Von Mises criterion. When the material harden during the plastic hardening, it becomes elastic in hydrostatic compression, which is in agreement with the experimental observations for the range of pressure considered here (Burlion 1997). It is important to notice that in our case the porosity will always decrease (Eq. 7). In traction where the plastic strains would be positive (and then produce an increase of the porosity) we use a damage model with an elastic threshold lower than the elastic strength for the Gurson’s criterion.
In order to have elastic properties that change with the evolution of the porosity, we use and homogenisation technique. The shear $G$ and bulk moduli $K$ are then functions of the material porosity $f^*$. These functions are computed using Mori-Tanaka (Mori and Tanaka 1973) homogenisation technique:

$$
\begin{align*}
K &= \frac{4K_M G_M}{4G_M + 3K_M f^*} \\
G &= \frac{G_M}{1 + \left(6K_M + 12G_M\right)\left(9K_M + 8G_M\right)f^*}
\end{align*}
$$

where $K_M$ and $G_M$ are the bulk and shear moduli of the material without pores respectively. We can notice that with these relations we have elastic properties that increase when the porosity decreases as experimentally observed (Burlion 1997).

As we saw in eq. (3), the viscoplastic strain are obtained following Perzyna’s approach. Colantonio and Stainier (1996) proposed a similar model in which the definition of the plastic multiplier accounts for the variation of porosity of the material. We follow here the same approach and define the viscoplastic multiplier as:

$$
\dot{\lambda} = \frac{f^*}{1 - f^*} \left(\frac{F_{NL}}{m_{\text{sp}}}\right)^{m_{\text{sp}}}
$$
where $m_{np}$ and $n_{np}$ are material parameters. The limiting case where the porosity tends to 1 (material failure) should not be considered because tension or compression damage reaches 1 before it may happen.

**Damage model**

The relation between the stress $\sigma_{ij}$ and the reversible strains $\epsilon_{ij}^e$ is given in equation (1). The bulk and shear moduli of concrete are assumed to remain constant in this section. Their variation, due to material compaction have been introduced before.

In Burlion et al. (2000), only one damage variable was introduced in order to capture the material response in tension. The corresponding response of the material in compression was far from being satisfactory (by simple coupling with plasticity). In order to have a better agreement with experimental data, we use the formulation due to Mazars that combines compression damage and tension damage. The combination of these two types of damage is given in the original work of Mazars (1984, 1986):

$$D = \alpha_c D_c + \alpha_t D_t$$  \hspace{1cm} (10)

where $D_c$ and $D_t$ represent the compressive and the tensile damage respectively while $\alpha_c$ and $\alpha_t$ are parameters corresponding to the loading path.

The growth of the two damage variables is governed by the elastic equivalent strain:

$$\tilde{\epsilon}^e = \sqrt{\sum_{i=1}^{3} \left( \epsilon_{ij}^e \right)^2}$$  \hspace{1cm} (11)

where $\epsilon_{ij}^e$ is the $i^{\text{th}}$ component of the tensor of the principal strains and $\langle x \rangle_s$ is the positive part of $x$.

The growth of $D_c$ and $D_t$ is defined by the following equations (Gatuingt and Pijaudier-Cabot 2002):

$$\dot{D}_i = \left( \frac{\tilde{\epsilon}^e - \epsilon_{D0} - \frac{1}{\alpha_i} \left( \frac{D_i}{1-D_i} \right)^{1/n_i}}{m_{Di}} \right)^{n_i}$$  \hspace{1cm} \text{for } i = c, t$$  \hspace{1cm} (12)
$m_{D_i}, n_{D_i}$ are material parameters that control the rate effect. $a_i, b_i$ are material parameters which govern the growth of damage in quasi-static tension and compression. $\epsilon_{D0}$ is the initial threshold of damage.

**Model Response**

The determination of the model parameters benefits from the fact that in uniaxial tension damage due to microcracking occurs without plastic strain growth and conversely, in compression dominated regime (with large enough confinement in order to avoid positive reversible strain) plastic strain growth is observed with the inherent variation of the porosity.

Overall, the constitutive relations contain two parameters that define the elastic behaviour (the shear and bulk moduli of concrete without voids), five parameters that control the material response in tension (including the damage threshold), four parameters which control damage growth in compression. Four parameters enter in the viscoplastic model and two enter in the equation that governs the variation of porosity, including the initial material porosity (the quantities $q_1, q_2, q_3$ assume usually fixed values). The initial porosity depends on the concrete mix. It is usually in the range of 0.3. The total number of model parameters might be considered to be quite high. In view of the three mechanisms described by the model and the various rate dependent effects, it seems difficult to arrive at a significantly smaller amount of parameters. Quasi-static experiments in tension, uniaxial compression and triaxial compression provide all the coefficients, except the parameters $m_{vp,t,c}$ and $n_{vp,t,c}$ (Burlion et al. 2000).

Figure 4: Model response for uniaxial tension test for several strain rates.
The determination of the remaining parameters is more intricate because it requires test data obtained for different strain rates. Such test data have been obtained for a single concrete mix. They include scabbing tests, split hopkinson tests on confined and unconfined specimens (Gatuingt and Pijaudier-Cabot 2002).

Figure 4 shows the response of the damage model in uniaxial tension tests carried out at various strain rates. The response is strongly dependent on the rate of stress, which is in agreement with experimental observations (Bailly 1999).

Figure 5 shows the response of the damage model in uniaxial compression. For the determination of the parameters we chose to have a behaviour in compression which is only slightly dependent on the strain rate. This is again quite consistent with test data. It should be pointed out, however, that the model response is substantially modified in compression, due to the coupling with a viscoplastic model. We can choose a set of parameters in order to have only the damage model in uniaxial compression with no irreversible strain. On the other hand, we can choose to have a coupled response (damage + plasticity).

Figure 6 shows the model response for a hydrostatic loading followed with a hydrostatic tension. We can see on this figure that we have a hardening behaviour of the concrete when the porosity decreases. We also can notice that for an unloading we obtain elastic properties greater than the initial ones (this is due to the reduction of porosity in the homogenized shear and bulk moduli). At least, when the stresses are positive, we obtain the rate dependent damage model. Static and dynamic loadings are computed in order to show the influence of the strain rate on the model response. This is on this figure that we can observe the effect of the Gurson’s plasticity as a Cap model for concrete.
Figure 6: Model response for triaxial compression with unloading for two strain rates

The model has been implemented in the finite element code LS-DYNA3D in an explicit format. We have implemented an explicit (Euler forward) integration scheme. All the incremental variables during the time increment $\Delta t$ starting at time $t$ are computed from the state variables evaluated at time $t$. The nonlinear response of the material is obtained by an explicit correction of the elastic prediction at each time steps (Burlion et al. 2000).

It is well known that explicit time integration is not accurate if the time increment is too large (i.e. if the viscoplastic incremental strains, damage or incremental porosity are too high). In the case of rate independent plasticity combined to damage, Burlion et al. (2000) have used the same integration scheme. It was observed that in practical cases where the three-dimensional finite element mesh is sufficiently fine in order to achieve an accurate description of the irreversible phenomena in the structure, the critical time step due to the explicit integration of the equations of motion is so small that the explicit integration of the constitutive relations is accurate enough. Given the fact that both the constitutive relations and the equations of motion are integrated explicitly, error accumulation might occur at these two levels. The balance between the internal, kinetic and dissipated energy is monitored in order to detect such situations. Finally, LS-DYNA3D has an erosion option that can be activated in the course of a numerical simulation in order to remove elements in which failure occurs. Without this technique, slab element directly in contact with the explosive are severely compressed. These highly deformed elements cannot flow around the explosive and the time step severely decrease. To compensate for this phenomenon in Lagrangian calculation, frequent re-zoning of the slab material in the neighbourhood of the explosive is required.
Numerical Simulations

In the following, we are going to discuss an example of numerical simulation. The foregoing results have been obtained within the framework of a research sponsored by Thomson Daimler Armements in France. For confidentiality reasons, the numerical values of the model parameters of the constitutive relations cannot be provided. Nevertheless, we will show a comparison between the computation and the experiment in order to illustrate the capabilities of the constitutive relations developed in the previous section. The main characteristics of this material have been provided in the previous section.

In order to evaluate the proposed model on a case representative of a real application, a test where an explosive is placed in contact with a concrete slab was carried out (Bailly 1999). For the sake of simplicity, the explosive as well as the concrete slab were axisymmetric. The circular concrete slab had a diameter of 1 meter and a thickness of 20 cm (Figure 7). These dimensions were such that the boundary conditions at the edge of the concrete slab did not influence the loading process, as the wave generated by the explosive travelled back and forth through the thickness of the slab. This test was instrumented by strain and pressure gauges embedded in concrete. Figure 6 and Figure 7 show the location of the gauges, the P2 plane corresponding to the mid-thickness of the slab.

Figure 7: Experimental device for the explosion in contact

Frequent re-zoning adds to the computation time and allows for variability in the results depending on how often and in what manner the analyst decides to perform the re-zoning operation. This element removal implies a loss of mass that is not very important in our problems. This erosion procedure can, for instance, be indexed on critical values of some internal variables in the model (Gatuingt and Pijaudier-Cabot 2002).
Figure 8: Finite element mesh used for the numerical simulation

Figure 8 shows the mesh used for the numerical simulations. The concrete slab is separated in three distinct parts in order to be able to refine the mesh on the level of the contact between the explosive and concrete.

Figure 9: a) Damage evolution and b) Porosity evolution in the concrete slab
Figure 10: Comparison between the simulation and the experiment: axial stress on the 3 planes versus time.

It is a 2D mesh, essentially for computational speed reasons, compared to a full 3D mesh (note that as axisymmetric mesh could have been used as well). The displacements perpendicular to the plane containing the finite element mesh are blocked. This condition represents the symmetry of revolution approximately. It should be pointed out that many problems were encountered in the numerical representation of the contact between the concrete slab and the explosive. A way of limiting these problems was to increase the number of element describing this contact significantly. This is the reason why a full 3D computation is not presented.

The equation of state used to model the detonation in the explosive is of the JWL type, which defines the pressure induced by the explosion. Detonation starts at the centre of the explosive. It generates a pressure wave that is transmitted to the slab through the contact conditions in between the two materials.

Figure 9a shows the evolution of the damage during the calculation. In this calculation, the criterion of erosion was also activated in order to visualise the scab that might be ejected from the back face of the slab. Although a significant mesh effect was observed at the interfaces between the regions of the finite element model where the element density is changed abruptly, the computed size of the scab is: diameter = 26 cm, depth = 4 cm. This is quite close to the experimental values measured on the slabs equipped with the embedded strain gauges.

Figure 10 shows the evolution of axial stress versus time for planes P1, P2 and P3. We can see that the model is also able to capture quite well the velocity and amplitude of the compression wave within the thickness of the slab.

Figure 9b shows the evolution of the porosity in the slab during the calculation. We can see on this figure that close to the explosive, where the spherical part of the stress field is the more important, we have a significant decrease of the porosity. Farther from the explosive, the energy due to the explosion is lower and the decrease of the porosity is less important. This effect can also be seen on Figure 11 that shows the evolution of the porosity versus time for planes P1, P2 and P3.
Conclusions

A constitutive relation aimed at describing the response of concrete in dynamics has been presented. It contains a description of microcracking in tension and compression, and of compaction due to hydrostatic compression. This model based on viscoplasticity coupled with a rate dependent damage model provides a CAP model for hydrostatic loadings. This cap obtained with a modified Gurson’s yield function and a homogenisation technique allows to describe the reduction of porosity that induces hardening of the material observed experimentally.

The prediction of the response of a concrete slab subjected to an explosion is quite consistent with the experiments, which lends some confidence in the capabilities of the proposed model at describing, with a sufficient amount of details, the response of concrete structures subjected to explosions and impacts.

As a final remark, it ought to be noted that the present approach does not account for the presence of interstitial water in concrete. The model has been calibrated with test data on the material not saturated with water (Humidity ≈ 60%). The influence of water should be expected on compaction and on the rate effect.

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References


